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Temperament and tuning of early 19th century Hispanic keyboard instruments: A study of the monochord integrated into a fortepiano made by Francisco Fernández (1828)

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Summary. The recovery of previously used tuning systems of musical instruments, led by the interpreters of historical repertoires, has widened our knowledge and our aesthetic perception of the sounds, harmonies, and repertoires of works from those times. For only a very few instruments have we been able to determine the original tones, whereas the mechanisms designed to tune keyboard instruments are of a remarkable reliability. Among the latter is the monochord integrated into the fortepiano made by Francisco Fernández in 1828. In this article, we evaluate the measurements, both physical and acoustic, of the tones of this device, and offer comparisons. Based on the conclusions of this analysis, we define a tuning system closely linked to another, contemporary one but with unique features that result in a number of sonorities perfectly adapted to the performance and aesthetics of the musical repertoires of Romanticism. Moreover, this system, which was probably used until the early 20th century, offers us a new harmonic coloring, one especially suited to the Iberian repertoire of the same time.

Keywords: Francisco Fernández (1766–1852) · Barcelona Music Museum · fortepiano · monochord · tuning · musical temperament · Romanticism

Resum. La recuperació dels sistemes d'afinació dels instruments musicals anteriors al sistema actual, liderada pels intèrprets dels repertoris històrics, ha eixamplat el nostre coneixement i la nostra percepció estètica dels sons, les harmonies i els repertoris procedents de les obres d'altres èpoques. Només hem estat capaços de determinar els sons originals de molt pocs instruments, mentre que els mecanismes ideats per afinar instruments de teclat són d'una fiabilitat considerable. Entre aquests últims hi ha el monocordi integrat al fortepiano de Francisco Fernández del 1828. En aquest article analitzem les mesures, físiques i acústiques, dels sons d'aquest aparell i oferim comparacions. Basant-nos en les conclusions d'aquesta anàlisi, hem definit un sistema d'afinació que té molta relació amb d'altres de contemporanis seus, però amb característiques pròpies que fan que gaudeixi d'unes sonoritats perfectament adaptades a la interpretació i l'estètica dels repertoris musicals del Romanticisme. Alhora aquest sistema, probablement emprat fins a principis del segle XX, ofereix un nou colorit harmònic, especialment adequat al repertori ibèric de la mateixa època.

Paraules clau: Francisco Fernández (1766–1852) · Museu de la Música de Barcelona · fortepiano · monocordi · afinació · temperament musical · Romanticisme

THE TUNING SYSTEM OF THE 1828 table fortepiano kept at Barcelona's Museu de la Música (catalogue no. MDMB 504) works by comparing the tones previously programmed on a monochord to those generated by the strings of the instrument. The monochord is a sonorous element that, although independent, has been integrated into the furniture and structure of the modern piano. It is an acoustic instrument designed to fix musical intervals based on geometric proportions, determined according to the different lengths of the strings in vibration. The name *mono* (one), *chordium* (string) comes from the fact that one works on a single string fixed on a resonance table, in which a number of positions and vibration lengths are chosen by sliding a moveable bridge, stopping it on the marked positions, which correspond to the piano notes.

From the theoretical analysis of the tones produced at each position of the cursor, we can determine and compare the fifth and third intervals, which constituted the fundamental basis of the tuning of the period. Thus, a first analysis arises from the physical measurements of the vibrating string at each segment fixed by the monochord. A parallel procedure is obtained by recording the tones produced by the string at each segment fixed by the position of the cursor. To catalogue the tones, in the winter of 2002 the monochord was tuned with the A of the first octave at 103.8 Hz, corresponding to the same note A of 415 Hz of two earlier registers. Next, a tone sample was recorded for each position of the cursor using three procedures of acoustic measurement. The set of these samples was used to establish the comparisons and valuations of the initial theoretical measurements.

From the analysis and the comparison of the results obtained from our study of the monochord, we were able to deduce the properties of a tuning system that was valid for the aesthetics of the musical repertoires of the end of the 18th century and beginning of the 19th century. This is especially interesting given the sonority of the works composed and interpreted at that time on the Iberian Peninsula.

History

The first descriptions of the monochord date back to the 5th century BC. The device was attributed to Pythagoras and it continued to be in use until the 19th century. In a number of medieval treatises it is referred to by its Latin name, as the *canon harmonicus*. In addition, it has been linked to the study of the historical ranges and tuning systems of that time. Among the medieval descriptions, the one that occurs most frequently is that of a table with a string supported by two fixed bridges and with a mobile bridge in the middle that, when slid in the manner of a cursor, cuts the string at the selected points to yield the tones of the different musical intervals. On the surface of the table, which measures 90–120 cm in length, a linear series of inscriptions and marks drawn with the help of a compass were used to determine the positions of the cursor for each musical interval. In the 12th century, the table was replaced by a resonance case, which improved the monochord's sonority.

During the Renaissance, from 1500 onwards, a new model was introduced in which the central cursor was no longer a feature; instead, one of the fixed bridges at the ends was mobile and assumed this function. This model was less precise, as the linear nature of the string was variable, resulting in a geometry that affected the regularity of the tension.

In the monochord attributed to Pythagoras, the tones were determined by the arithmetic proportions among the vibrant lengths of the string. These tones are considered the main intervals of the Pythagorean range or tuning system.

In the geometric calculation of the musical intervals, the different tones could be obtained through two methods of dividing the string, which provided the higher and the lower ranges with respect to the initial one. For instance, a tone two octaves higher than D or E, or two octaves lower than C could be produced by dividing, by means of a compass, the whole string or only one section of it into nine equal parts (Fig. 1).

The positioning of more complicated intervals, such as the Pythagorean semitone, was made possible with the monochord described by Odo of Saint Maurus around the year 1000, at the time of Pope Sylvester II, who authorized its scientific use. This model established the operational basis of Renaissance instruments, which rely on the ascending division, as it proved more practical for the musical language of the time. Even so, the monochord would continue to provide a meeting point between scientific reasoning, in the demonstrative field of proportions, and musical practice, in its need to define the ranges and proportions among harmonic sounds.

In studying and describing the fortepiano monochord of Francisco Fernández (1828), we incorporated physical and musical measurements. Our first measurements of the marks and inscriptions on the wooden support of the string were carried out in December 2002, with the collaboration

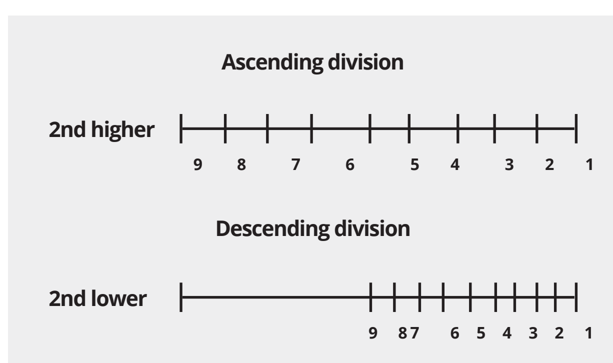

Fig. 1. Methods for the division of the monochord string.

of Joan Pellisa. These measurements were later checked against those taken in August 2010 by Manel Barcons. The reference to the octaves, by means of the subindices –1, 1, 2, 3 and 4, were placed from the A3 corresponding to the 440 Hz *A* sound.

The fortepiano and its maker. The fortepiano preserved at Barcelona's Museu de la Música (Fig. 2A) was built by Francisco Fernández, a relative of Diego José Fernández, an instrument maker born in Vera (Almería, Spain), as detailed in an inscription on a frontal plaque (Fig. 2B). The instrument is of the small-case type intended for domestic use, with an area of 132.7 × 49.3 cm and a height of 22.4 cm. It is supported by four fixed legs of lathed wood. Its keyboard has 73 keys, from F –1 to F 6, spanning a width of 91.8 cm. From the keyboard, a simple English-type device is operated and acts on the 73 individual metal strings. The strings are laid out in an oblique shape on the soundboard, made of fir, and show cross-wise streaking. They are made of different materials: steel and copper for the central strings, steel for those that are high-pitched, and torched brass for those of low-pitch.

The monochord is integrated inside the cabinet, mounted on a long and resistant piece of wood that is fixed on the front part of the soundboard. On the front central part of the keyboard, under the lower panels of the pinewood case, the traction of two pedals is fixed, mounted on a lyre-shaped support. The pedal on the right activates the monochord and that on the left frees the dampers. The frontispiece of the piano, over the keyboard, which is curved on its ends, is decorated with latticed wood, matching the two triangular lids of the top side drawers located on both sides of the keyboard (Fig. 2C). This instrument, from the Folch i Torres-Baget collection, entered the Museum in 1947.

Francisco Fernández (1766–1852) was born in Asturias, Spain, and moved to Madrid between 1792 and 1799. In 1827, he was granted the title of *Honorario constructor* as purveyor of musical instruments to the Spanish Royal Household. His Madrid workshop had several locations: Corredera de San Pablo, 20 in 1799; Calle del Barquillo, in 1804; Calle de San Fernando, 9 in 1827; Calle de San Fernando, 5 in 1828. In 1817 he founded a piano-making school, where students would also learn languages.

The construction of the piano in the museum's collection reflects Fernández's many years of experience. He was 62 and his workshop was located in Calle de San Fernando 5. We therefore think that this instrument incoporates all of Fernández's professional abilities. We do not have sufficient documentation to ascertain whether the incorporation of the monochord came through the influence and knowledge of similar devices, such as the "tuning machine for harpsichords, piano-fortes, organs, guitars" of W. Thomson of 1787, with a patent commercialized by Longman & Broderip [7]. The latter device had a mechanism to drag the cursor, another one for its fixation, and a lever to activate the hammer.

Discovery and description of the monochord. The first reference to the discovery of the tuning mechanism of this fortepiano is an entry in the catalogue of Barcelona's Museu de la Música [8, p. 225], in the description of the instrument. It is a special characteristic that has always been stressed in the other publications of the museum as well. On a later occasion, Cristina Bordas mentioned it in one of her texts as an "*artilugio para afinar*" (a tuning mechanism) [5,6].

There have been only a few descriptions of the use of the monochord in the process of piano tuning since the 19th century. In Spain, Felip Pedrell, in his *Diccionario Técnico de la Música*, offered the following description of one of these devices, attributing its invention to a certain Baller (sic): "Cronómetro-monocordio. Aparato destinado á afinar los pianos, que resonaba por medio de un teclado. Fué inventado en 1827 por Baller, fabricante de pianos." (Chronometer-monochord. A device used to tune pianos that resounded by means of a keyboard. It was invented by Baller, piano maker, in 1827). [9] The date of the invention is surprising, as it is a year before the construction of our fortepiano. It would be interesting to investigate the probable relationship between the two devices.

This monochord was precisely integrated into the inside of the fortepiano's case, and although it is in fact inde-

A B C

Fig. 2. (**A**) Fortepiano made by Francisco Fernández. Museu de la Música, Barcelona. (**B**) Plaque on the F. Fernández fortepiano's front with the inscription "FERNÁNDEZ Constructor de Pianos / DE LA RL. CAMARA DE SS. MM. / Calle de Sn. Fernando N.° 5 / año de 1828 / Madrid". (**C**) Monochord. Frontpiece of the keyboard with the top part decorated with latticed wood.

pendent of the piano, it makes use of the latter's resonance to strengthen the volume of the sounds it produces. It is fixed over the keyboard, just behind the front part and perpendicular to the direction of the keys. When the lid is down, it remains hidden and it is held by means of metal bolts over the soundboard, with which it couples its vibrations. It is made of a rectangular compact piece of fine-grained wood, possibly pear wood, that is 118.8 cm long and has a rectangular section of 27 × 37 mm. The narrower surface corresponds to the upper side whereas on the rear side, which looks onto the strings, there is a groove, 13 mm wide and 9 mm deep, that holds a brass string. On the left side, the string is secured with a tack and on the right, over a metal tuning pin. The string begins to vibrate by means of the movement of a brass plectrum mounted over a jujube hammer that stands perpendicular to the string, resembling that of a harpsichord, but without the damper. This hammer, which is 15 mm wide, runs inside a perforated slit located 83 mm from the extreme of the wood strip (Fig. 3A).

The vibrating length of each note of the string remains set between the position of a fixed metal capo, placed between the tuning pin and the plectrum at 25 mm from the right end, and the position of the metal capo nailed to the right of the base of a moveable cursor. This cursor, resting on the surface of the upper plane of the monochord, is made of the same wood and stands under the tension of the string by means of two metal hooks nailed to the base; one of them, as noted before, acts as a capo (Fig. 3A). Its movement along the string, eased by treating the wood with talcum powder, allows it to be placed in fixed positions, marked on the upper face with lines that are perpendicular to the string and with successive numbers. This placement determines the intonation of the different notes, which become the intonation references for the corresponding fortepiano strings. These positions also correspond to those of the lines and small holes marked next to the groove. It is very probable that the holes were used to insert blocks to fix the cursor at each position (Fig. 3B,3C). The geometric diagram of the position of the string corresponds to the one described in Fig. 3D.

The first position of the cursor is marked with a square box with two diagonal lines, where the cursor has to be placed in order to produce the first note. The marks and inscriptions indicating the successive positions of the cursor consist of perpendicular engraved lines numbered from 17 to 41 and coinciding with the right end of the cursor (Fig. 4). Under the numbering at each position of the cursor, there are other numbers and symbols, not always easy

A B C D

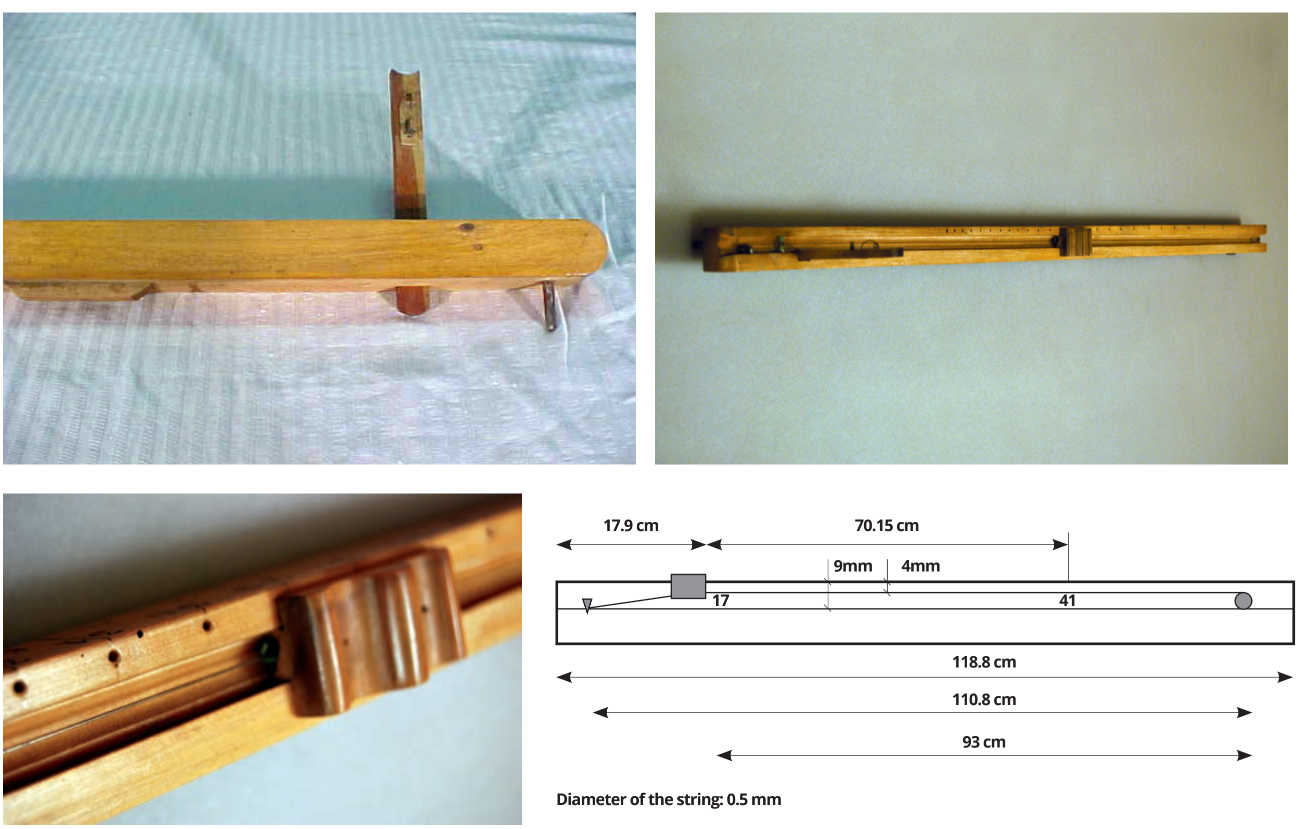

Fig. 3. (**A**) Brass plectrum that, when displaced, sets the string in movement. (**B**) Cursor that works as a capo. (**C**) Mobile cursor. The lines and holes made close to the groove. (**D**) Geometry of the string.

to identify, indicating the musical notes at each point. On the front face of the monochord, the one that remains hidden by the front of the keyboard, we find other symbols indicating the musical notation of each position of the cursor, marked with letters, and an inscription written in pencil that reads "Fernando Baranda" (Fig. 4D and Table 1).

Movement of the hammer and the plectrum is activated from the right pedal, from which a system of transmission levers originates. A spring guarantees the return of the hammer to the at-rest position (Fig. 5).

Use of the monochord to tune the piano

The tones of the monochord, once the reference note A has been tuned, are produced by using the pedal on the right, and are selected through the different positions of the cursor. The strings of the piano are tuned with a special key that turns the wrest pins, and are made to sound from the keyboard until exact unisons are obtained, with no interfering beats, taking the monochord as a reference. For each position number of the cursor there is a corresponding string with the same number indicated on a wooden plaque attached next to the wrest plank (Fig. 6).

Even so, it is not possible to tune all the strings through comparison with the monochord, as the piano has 73 strings and keys, from F–1 to F6, and the cursor only determines 24 positions. Therefore, 49 notes have to be tuned either through a comparison with their monochord octaves or with the notes of other octave of the piano.

Table 1. Symbols and notes corresponding to the positions of the cursor

Position of the cursor (number)	Symbols on the upper face	Symbols on the rear face	True notes
17	A	illegible	A1
18	B	a#	A#/Bb
19	0	B	B
20	#	C	C2
21	D	C#	C#/Db
22	x	D	D
23	3	D#	D#/Eb
24	Λ	E	E
25	F	F	F
26	#	#	F#/Gb
27	A	g	G
28	#	illegible	G#/Ab
29	B	illegible	A2
30	H	illegible	A#/Bb
31	6	illegible	B
32	#	illegible	C3
33	D	illegible	C#/Db
34	≠	D#	D
35	S	E	D#/Eb
36	J	F	E
37	≠	illegible	F
38	3	g	F#/Gb
39	S	g#	G
40	A	a	G#/Ab
41	blank space		A3

A B C D

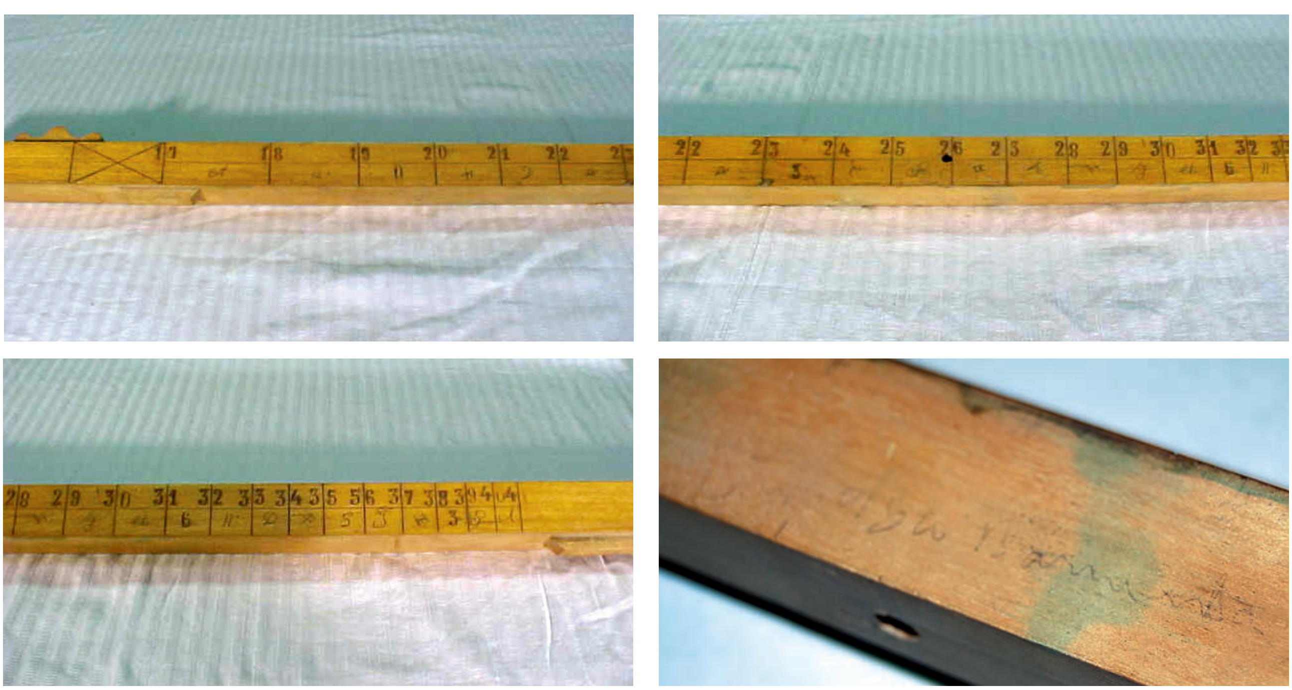

Fig. 4. Upper side of the monochord. (**A**) Left. (**B**) Middle. (**C**) Right. (**D**) Front face with the inscription "Fernando Barana".

A

B

C

Fig. 5. View of the elements of the piano. (**A**) Pedals. (**B**) Transmission. (**C**) Puller of the plectrum.

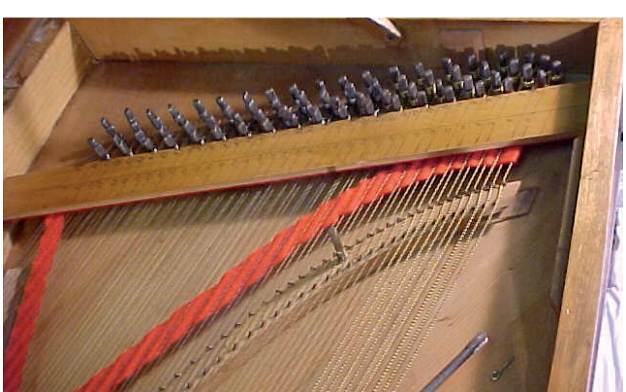

Fig. 6. Numbering of the strings.

The notes and intervals of the monochord

Measuring the length of vibration of each note. The sequence of positions of the monochord cursor, which are marked, ruler-like, on the upper surface, determines several acoustic lengths of the string, which, once launched into vibration by means of the plectrum, yields different reference sounds. The theoretical acoustic length (L) is therefore delimited by the distance between the tuning pin of the string, on the right, and the moveable capo, which holds the string from the cursor itself, placed in such a way that its right end coincides with the numbered lines on the upper surface of the monochord. For each theoretical length, measured over the numbered position markings, a small shortening of 1.5 mm has to be taken into account, owing to the fact that the metal hook fixed to the cursor works as a capo and stands out from the edge of this piece. At each position of the cursor, a block would have been inserted in a small hole on the inner face of the monochord, the same face over which the cursor runs. That a block was used is certain, given the considerable wear along the rims of the holes. The block's thickness could have offset the sliding of the vibrating length due to the position of the capo. Thus, there is no need to correct the reading of the theoretical L if it is measured directly from the finer lines and small holes of the inner face of the monochord.

In view of all these factors, the measurement of each acoustic length (L) is fixed through the measurements taken over the marks of the ruler, applying to it a correction of 1.5 mm (L_1). These marks determine 24 positions. The first position, number 17, corresponds to the second A of the keyboard, and the last position, number 41, to the fourth A. Each stopping point of the cursor is placed at a distance of one semitone from the following one, and the whole itinerary corresponds to the complete series of semitones that comprise the division of two octaves.

If the theoretical length for each position is measured in accordance with the procedure detailed above, the measurements of all the semitones that make up the three octaves can be obtained. As a first measurement unit, we have chosen the *cent*, because among the logarithmic units used in the measurement of intervals it is the one used on a standard basis by analysts of historical tuning systems and by practicing musicians using temperaments of the period. The number of cents in an interval is given by the formula

$$n \text{ (cents)} = \log (L_1/L_2) \times 1200/\log 2$$

where L_1 and L_2 are the two acoustic lengths, L_2 being the minor length and L_1 the major one, corresponding to that of the previous mark, placed at the distance of one semitone.

Applying this calculation to all L_1 corresponding to the different positions on the cursor yields all the measurements of the semitones on the ruler. Table 2 shows the theoretical length of the string for each position on the cursor. From this length, the measurements of the semitone intervals and their corresponding deviations with respect to the temperate semitone of 100 cents can be deduced. We ignore the precision of these tenths-of-a-millimeter measurements of the lengths of the vibrant string segment. As we will see later, the effect on the acoustic quality of the differences does not substantially alter the analysis of the tuning system nor the tuning of the piano. We must also consider the deformations of the ruler with respect to the original construction measurements.

We compared the results of a 1-mm variation in the precision of the measurement of the lengths of the string segments. The differences in positive and negative deviations were between 3 and 5 cents (data not shown), i.e., an almost imperceptible effect on the acoustic quality of the different intervals that we compared in our analysis of the tuning system. We also measured—individually and successively—the series of semitones, adding them to calculate the intervals that they determine. The results are modulated by the accumulation of partial errors arising from the individual measurements (data not shown). We therefore discarded this method.

Irregularities in the octaves. To determine the general reliability of the measurements, we checked the octaves generated by the positions of the cursor with respect to those that would be obtain using the theoretical length of the string. The "A" mark that would form the first octave is located 4.5 mm further to the left of its theoretical position, which can be calculated by dividing by two the L of the first A. In other words, there has been a small contraction in this segment. This can be explained by a longitudinal deformation of the monochord, owing to the loss of mass during aging of the wood and the elastic expansion of the opposite end. We tested the exactitude of other octaves in order to find the point at which these errors might affect the reliability of the measurements. In Table 3, for each position L, a of the cursor and L,b of its octave, we calculated the theoretical length of the string (theoretical b). The difference, in mm, is the error in each position, which we also took into account as a correction per semitone and per fifths, considering it applicable to the different measurement zones of the monochord.

In conclusion, the distribution of these systematic errors among the 12 semitones of the octave, on the order of tenths, did not noticeably affect measurements of the semitone intervals. For calculation of the fifths, especially in measuring the notes of the central area, from F2 downward, a correction factor from –10 to 15 cents was introduced, which was offset by corrections of some +10 cents at the ends of the scale of the wood strip.

Table 2. Measurement of the semitones

Position of the cursor	Key and string	Acoustic L (mm)	L2 = L-d (mm)	Cent semitones	Deviation with respect to the temperate semitone
17	A1	943.0	941.5		
18	A#/Bb	876.0	874.5	127.8	27.8
19	B	822.0	820.5	110.3	10.3
20	C2	775.0	773.5	102.1	2.1
21	C#/Db	734.0	732.5	94.2	–5.7
22	D	697.0	695.5	89.7	–10.2
23	D#/Eb	654.0	652.5	110.4	10.4
24	E	616.0	614.5	103.8	3.8
25	F	586.0	584.5	86.6	–13.3
26	F#/Gb	554.0	552.5	97.4	–2.5
27	G	526.0	524.5	90.0	–9.9
28	G#/Ab	493.0	491.5	112.5	12.5
29	A2	467.0	465.5	94.0	–5.9
30	A#/Bb	442.0	440.5	95.5	–4.4
31	B	416.0	414.5	105.3	5.3
32	C3	393.0	391.5	98.8	–1.1
33	C#/Db	372.0	370.5	95.4	–4.5
34	D	354.0	352.5	86.2	–13.7
35	D#/Eb	336.0	334.5	90.7	–9.2
36	E	317.0	315.5	101.2	1.2
37	F	297.0	295.5	113.3	13.3
38	F#/Gb	280.0	278.5	102.5	2.5
39	G	264.0	262.5	102.4	2.4
40		250.0	248.5	94.8	–5.1
41		237.0	235.5	93.0	–6.9

Sound imprecision: geometry and inharmonicity of the string. There are other sources of imprecision of each note's resulting tone with respect to the theoretical positions marked on the monochord. They are due to variation in the tension of the string, depending on the closer or more distant position of the cursor to the ends where the string is fixed, and to the actual inharmonicity of the string, which arises from irregularities in both the material and the parts of the different sections, as shown in Figure 8.

General sound effect. Finally, note that the second A of the monochord (Table 3), corresponding to note 29 of the keyboard, has a length of 467 mm, closer to the lengths of the string of the piano's A_3, which is 300 mm, and to that of other instruments of the same period, which tend to be around 400 mm. This measurement places us in a sphere of timbric perception in which the monochord seems to be tuned an octave

Table 3. Irregularities in the octaves

(a–b)	L, a (mm)	L, b (mm)	Theoretical b (mm)	Difference (mm)	Difference (cents)	Excess	Correctionby semitone	Correction by fifth
A1–A2	943.0	467.0	471.5	4.5	16.60	short	1.4	11.1
A2–A3	467.0	232.0	233.5	1.5	11.16	short	0.93	7.4
G2–G3	526.0	264.0	263.0	–1.0	–6.57	long	–0.5	–4.4
F2–F3	586.0	297.0	293.0	–4.0	–23.47	long	–2.0	–15.6
D2–D3	697.0	354.0	348.5	–5.5	–27.11	long	–2.3	–18.1
C2–C3	775.0	393.0	387.5	–5.5	–24.40	long	–2.0	–16.3

Table 4. Comparison between real and theoretical measurements

L2 fifths	Cents	Deviation with respect to the perfect fifth	Correction +10 cents
C2–G2	688.8	–13.2	–3.2
G2–D3	685.6	–16.4	–6.4
D2–A2	693.3	–8.7	1.3
A2–E3	670.7	–31.3	–21.3
E2–B2	679.6	–22.4	–12.4
B1–F#2	683.1	–18.9	–8.9
F#2–C#3	689.5	–12.5	–2.5
C#2–G#2	689.0	–13.0	–3.0
G#2–D#3	663.8	–38.2	–28.2
D#2–A#2	678.3	–23.7	–13.7
A#1–F2	696.0	–6.0	4.0
F2–C3	691.6	–10.4	–0.4

Table 5. Measurements of the major third intervals

Major thirds	Cents	Deviation with respect to a perfect third	Correction -10 cents	Type of third[a]
C–E	398.4	12.4	2.4	A
D–F#	398.5	12.5	2.5	A
E–G#	386.7	0.7	–9.3	B
F–A	394.1	8.1	–1.9	A
G–B	407.5	21.5	11.5	C
A–C#	395.2	9.2	–0.8	A
Bb–D	385.8	–0.2	–10.2	B

[a]A, between –2 and 5; B, <–5; and C, >5

lower than the piano. But this difference in octaves does not affect or hinder the final result of the tuning process.

Calculations with the lineal measurements of the cursor. Referring again to the semitone intervals listed in Table 2, we can see that the tuning system consists of fairly different semitones, with notable deviations, with respect to the 100 cents of the same temperate semitone. These deviations range from +10.0 to –13.7 cents, and exceptionally, 27.8 cents in the case of the first semitone.

To identify the tuning system of the monochord, we compared the true fifths, obtained through the different positions of the cursor, with the perfect fifths of 702 cents—which would be obtained as perfect fifths calculated from the same initial positions—by using the previously stated formula. The comparison between the true fifths and the theoretical ones is shown in Table 4. The cyclical order used in the table resembles the one that tuners use.

In Table 4, the deviations were calculated with respect to the perfect fifth of 702 cents, and the final column shows the deviations through the correction factor suggested above, which would correct both the deformation of the scale and the change in tension of the string through sliding of the cursor. Nonetheless, we calculated that this correction only affects the real tuning by 1 Hz.

The comparison between real and theoretical measurements (Table 4) led us to deduce that the set of fifths that would be obtained with the monochord can be divided into three groups: eight similar ones (A), which are close to perfect fifths (< 10 cents), specifically those formed over C, C#, D, F, F#, G, A#, B; three corresponding to A, E, G# and D# are shorter, from 10 to 20 cents (B); and one that is formed over G#, which is even shorter, with a difference of more than 25 cents.

The thirds

Using a procedure similar to the one used to compare the fifth intervals, we analyzed the major third intervals of the main chords of the time, thereby obtaining greater insight into the tuning system. Table 5 shows the measurements of these intervals formed over the diatonic notes, from C to Bb, with the corresponding measurement in cents. The third column shows the deviations with respect to the perfect major third of 386 cents and to the fourth, with their values amended using the same factor applied to the fifths. The final column shows the valuations with respect to the deviations, grouped into three categories.

As seen in the table, among the measured major thirds, four from group A come close to perfect major thirds. They are those formed over C, D, F and A. There are also two shorter ones(<5 cents), over E and Bb, and a longer one (> 10 cents) that forms over the G.

Acoustic analysis of the sounds of the monochord. In winter 2002, we tuned the monochord with the A of the first octave at 103.8 Hz, corresponding to the same A note of 415 Hz of two registers higher. Next, we determined a tone sample for each position on the cursor, using three acoustic measurement procedures. This set of samples was used to establish the comparisons and valuations of the initial theoretical measurements.

The first spectral analysis was carried out using the Spectrum Analyzer application from the software of the professional Samplitude 7.12 (VIP file, Fernández At02, and in a WAV file, named Fernández At02). The spectral graph of the first sound of the monochord, corresponding to position 17 on the cursor and to note A1 on the keyboard, is shown in Fig. 7. The graph has a first dominant frequency at 107.0 Hz, with a first partial at 301.0 Hz, a note that would correspond to the keyboard's A1. This second peak gives us the essentials of the tones that we hear at each position of the cursor.

To obtain greater precision, these measurements were checked against those made with another analyzer, the Sonic Visualizer 1.7.2 (Fig. 8). The measurements of the essentials, given in Hz, for each position on the cursor, obtained with both types of software, are shown in Table 6.

The differences in the results of the two analyses are remarkable, with deviations ranging from –13.3 to +11.1 Hz. Negative deviations were more frequent. Moreover, it should be noted that the same deviations were not maintained between identical notes on different octaves. The acoustic and electronic elements used probably introduced mechanical, acoustic, and sensitivity type errors. Considering these analytical measurements as not being very precise and given the margin of error that they presented, we used yet another method to measure the frequency, comparing those results with the others. In this new method, we analyzed the sounds registered with an electronic tuner, measuring them with the deviation quadrant with respect to the note tuned in identical temperament.

Measurements with an electronic tuner

By using the electronic Zen-On Quartz Chromatina tuner, we registered readings with reference to the fixed notes of the temperate tuning of the A at 415 Hz. Table 7 shows the results of the two methods and the deviations in cents with respect to the closer temperate note (column 3). From the

Table 6. Measurements of the essentials in Hz with two types of software: Spectrum Analyzer and Sonic Visualizer

Note	Spectrum Analyzer	Sonic Visualizer	Differences between spectra
A1	107.5	107.7	0.2
A#/Bb	109.5	110.0	0.5
B	128.0	116.0	–12.0
C2	129.5	128.0	–1.5
C#/Db	131.0	132.0	1.0
D	139.5	137.0	–2.5
D#/Eb	148.6	149.0	0.4
E	149.1	153.0	3.9
F	151.3	162.4	11.1
F#/Gb	171.9	171.0	–0.9
G	173.0	179.0	6.0
G#/Ab	194.0	194.0	0.0
A2	215.3	202.0	–13.3
A#/Bb	215.4	220.0	4.6
B	237.1	230.0	–7.1
C3	258.0	246.0	–12.0
C#/Db	258.1	257.0	–1.1
D	280.2	274.0	–6.2
D#/Eb	302.0	287.0	–15.0
E	322.0	310.0	–12.0
F	344.0	331.0	–13 0
F#/Gb	343.0	349.0	6.0
G	365.2	369.0	3.8
G#/Ab	388.2	394.0	5.8
A3	428.5	421.0	–7.5

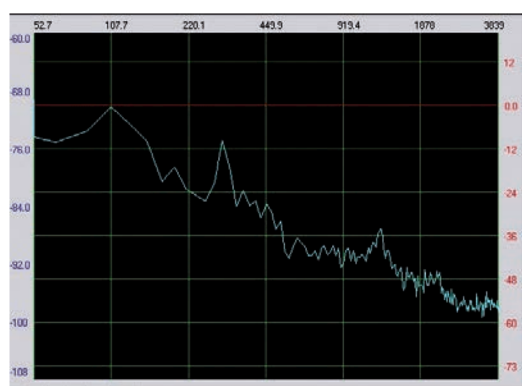

Fig. 7. Spectral graph of the monochord.

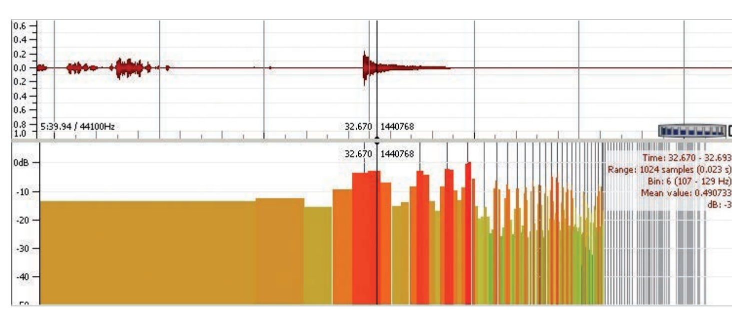

Fig. 8. Measurements of the spectral graph as analyzed using Sonic Visualizer 1.7.2.

Table 7. Comprison of the deviations in cents with respect to the nearest note, obtained by using two measuring instruments

Position on cursor	Key	F1 temperate at 415 Hz	Deviation from reading (cents)	Log F2 at 415 Hz	F2 Zen-On (Hz)	Sonic Visualizer	Difference between the two (Hz)
	G#	98.0					
17	A1	103.8	0	2.0163	103.8	107.7	3.9
18	A#/Bb	110.0	15	2.0376	109.1	110.0	0.9
19	B	116.5	30	2.0590	114.5	116.0	1.5
20	C2	123.5	30	2.0840	121.3	128.0	6.7
21	C#/Db	130.8	25	2.1104	128.9	132.0	3.1
22	D	138.6	5	2.1405	138.2	137.0	–1.2
23	D#/Eb	146.8	25	2.1605	144.7	149.0	4.3
24	E	155.6	15	2.1881	154.2	153.0	–1.2
25	F	164.8	10	2.2145	163.9	162.4	–1.5
26	F#/Gb	174.6	3	2.2413	174.3	171.0	–3.3
27	G	185.0	–7	2.2689	185.7	179.0	–6.7
28	G#/Ab	196.0	10	2.2897	194.9	194.0	–0.9
29	A2	207.7	2	2.3168	207.4	202.0	–5.4
30	A#/Bb	220.0	10	2.3399	218.7	220.0	1.3
31	B	233.1	10	2.3650	231.7	230.0	–1.7
32	C3	246.9	10	2.3901	245.5	246.0	0.5
33	C#/Db	261.6	3	2.4169	261.2	257.0	–4.2
34	D	277.2	–5	2.4440	278.0	274.0	–4.0
35	D#/Eb	293.7	–12	2.4709	295.7	287.0	–8.7
36	E	311.1	–5	2.4942	312.0	310.0	–2.0
37	F	329.6	0	2.5180	329.6	331.0	1.4
38	F#/Gb	349.2	–10	2.5456	351.3	349.0	–2.3
39	G	370.0	15	2.5644	366.8	369.0	2.2
40	G#/Ab	392.0	12	2.5903	389.3	394.0	4.7
41	A3	415.3	5	2.6171	414.1	421.0	6.9

ratio of this measurement and the frequency of the temperate tuning of the lower semitone (F1), we calculated the true frequency of the note measured (F2), since the number of cents (c) in the interval between two notes with frequencies F1 and F2 is:

$$c = 3986.3 \times \log (F2/F1)$$

where F1 is the lower note. If we set F1 as a frequency, in equal temperate scale, of the note placed a semitone lower than F2, measured with the Zen-On tuner, over an A of 415 and define 'd' as the deviation of F2 with respect to the corresponding temperate frequency, the frequency of each note from the reading of the electronic tuner can be obtained. Below, we detail the six steps of the process following that of the A; these would repeat themselves in parallel form for each of the 40 remaining notes. For example:

[1]The F2 of the A (approx. 415 Hz) has a deviation of 10 cents over the exactly tempered tuning, therefore, d = 10.
[2] F1 of the temperate G# = 392 Hz
[3] If k = 3986.3
[4] 100–d = k × (log F2 – log F1)
[5] log F2 = (100–d) / k + log 392 = 2.6171
[6] F2 = antilog (log F2) = 414.1

A comparison of the values of the frequencies measured with the Zen-On and the results of the spectral analysis of the Sonic Visualizer showed deviations of ±7, which is more acceptable than the deviations obtained when the Spectrum Analyzer from Samplitude was used. The differences between the theoretical frequencies derived from the marks of the monochord and the results measured with the Zen-On were minimal, with deviations from –4.7 to 2.2 Hz. Moreover, they were not excessively different from the variations between the results from the Zen-On and the Sonic Visualizer, of ∀ 7 Hz, as seen in Table 7, which compares the frequencies obtained with the three measuring methods. Therefore, in the following, to calculate the intervals of fifths and in the subsequent comparisons, we use the data obtained with the linear and acoustic measurements of the Zen-On.

Table 8 shows the measurements of the semitones formed with the note immediately higher (F2), from the linear measurements of the monochord, obtained from the

Table 8. Measurements of the semitones estimated from the readings of the electronic tuner

Cursor	Acoustic L 2 mm	Key	Cents of the semitone	F1 ZenOn	F2 in Hz	F2 at 415 Hz	Difference between the two F2 values
17	943	A2		103.8	103.8	103.8	0.0
18	876	A#/Bb	127.6	109.1	111.7	110.0	1.7
19	822	B	110.2	114.5	116.2	116.5	–0.3
20	775	C3	101.9	121.3	121.5	123.5	–2.0
21	734	C#/Db	94.1	128.9	128.1	130.8	–2.7
22	697	D	89.5	138.2	135.8	138.6	–2.8
23	654	D#/Eb	110.2	144.7	147.3	146.8	0.4
24	616	E	103.6	154.2	153.7	155.6	–1.9
25	586	F	86.4	163.9	162.1	164.8	–2.7
26	554	F#/Gb	97.2	174.3	173.3	174.6	–1.3
27	526	G	89.8	185.7	183.6	185.0	–1.4
28	493	G#/Ab	112.2	194.9	198.2	196.0	2.2
29	467	A3	93.8	207.4	205.7	207.7	–1.9
30	442	A#/Bb	95.3	218.7	219.1	220.0	–0.9
31	416	B	105.0	231.7	232.4	233.1	–0.7
32	393	C4	98.5	245.5	245.3	246.9	–1.6
33	372	C#/Db	95.1	261.2	259.4	261.6	–2.3
34	354	D	85.9	278.0	274.5	277.2	–2.7
35	336	D#/Eb	90.3	295.7	292.9	293.7	–0.8
36	317	E	100.8	312.0	313.4	311.1	2.3
37	297	F	112.8	329.6	333.0	329.6	3.4
38	280	F#/Gb	102.0	351.3	349.6	349.2	0.4
39	264	G	101.9	366.8	372.5	370.0	2.6
40	250	G#/Ab	94.3	389.3	387.3	392.0	–4.7
41	237	A4	92.4	414.1	410.6	415.3	–4.7

reading of the Zen-On electronic tuner. F1 is the frequency of the lower note, measured with the Zen-On tuner. As stated above, c = 3986.3 × log (F2/F1). Thus, the frequency of the sound at a semitone higher is F2 = antilog (R), where R = c/3986.3 + log (F1).

Comparison of the different measurement methods

Comparison of the intervals of fifths. The next step was to compare the fifths generated from the measurement of the samples of real sound registered, measured with the Sonic Visualizer, with those calculated from the readings by the Zen-On electronic tuner.

Table 4 showed the deviation of each fifth, calculated from the vibrating length of the string (L2), with respect to perfect tuning (702.0 cents), and the values more approximate to the corrections we established earlier. Given the differences among some of the fifths based on diatonic notes, we decided to compare them with the measurements of the same intervals in other registers. In the cases in which such measurements were possible, we obtained the results shown in Table 9. The fifths were compared on the basis of the values obtained through the values of the vibrating length (L2) of the string and those from the Zen-On tuner.

As seen in Table 9, almost all the fifths were similar, with the exception of those formed over notes A1 and A2, which differed by 66.5 cents, a little more than a quarter of a tone, while the fifth formed over A2 was wider than the fifth formed over A1. The positions of both As were very similar to the distance of an octave: 943.0 cents from A1 to A2 (see Table 3). Theoretically, A2 should be at 471.5 but it occurred at 476.0, a difference of 4.5 cents.

If we compare the octave of the E, from E2 to E3, there are 616.0 cents; theoretically, E3 should be at 308.0 rather than at 317.0. The difference, therefore, is only 9 cents, which is not sufficiently larger than the different results in the fifths. We therefore re-calculated the fifths, adding the seven semitones that make them up, which yielded: (a) for the A1–E2 fifth, 737.1 cents, and (b) for the A2–E3 fifth, 670.7 cents. The difference of 66.4 cents that separates them does not differ significantly from 63.8 cents. Consequently, if we were to choose between the longest and the shortest differences, we would choose that of 670.7 cents, because the

Table 9. Comparison between L2 and Zen-On

Fifths I L2	Cents I	Fifths II Zen-On	Cents II	Deviation I (exact tuning)	Deviation II (exact cents)	Difference between I and II (cents)
C2–G2	688.8	C3–G3	688.8	–13.2	–13.2	0.0
D2–A2	693.3	D3–A3	694.6	–8.7	–7.4	1.4
A2–E3	670.7	A1–E2	737.2	–31.3	35.2	66.5
B1–F#2	683.1	B2–F#3	683.1	–18.9	–18.9	0.0
C#2–G#2	689.0	C#3–G#3	688.0	–13.0	–14.0	–1.0
A#1–F2	696.0	A#2–F3	688.3	–6.0	–13.7	–7.7

Table 10. Quality of the fifths (L)

Fifths L2 (vibrating length)	Cent	Exact deviation	Correction +10 cents	Assessment
C2–G2	688.8	–13.2	–3.2	A
G2–D3	685.6	–16.4	–6.4	A
D2–A2	693.3	–8.7	1.3	A
A2–E3	670.7	–31.3	–21.3	C
E2–B2	679.6	–22.4	–12.4	B
B1–F#2	683.1	–18.9	–8.9	A
F#2–C#3	689.5	–12.5	–2.5	A
C#2–G#2	689.0	–13.0	–3.0	A
G#2–D#3	663.8	–38.2	–28.2	C
D#2–A#2	678.3	–23.7	–13.7	A
A#1–F2	696.0	–6.0	4.0	A
F2–C3	691.6	–10.4	–0.4	A

Table 11. Deviations of the fifths determined by Zen-On (ZO)

Fifths	ZO	ZO – exact 702 cents	Assessment
C–G	695.0	–7.0	A
G–D	698.1	–3.9	A
D–A	703.1	1.1	A
A–E	706.9	4.9	A
E–B	705.0	3.0	A
B–F#	720.0	18.0	B
F#–C#	700.0	–2.0	A
C#–G#	691.0	–11.0	B
G#–D#	722.0	20.0	C
D#–A#	715.0	13.0	B
A#–F	710.0	8.0	B
F–C	700.0	-2.0	A

Table 12. Assessment of the deviations of the fifths (in cents)

Fifths	Deviation from the actual (linear)	Deviation from the actual (ZO)	Difference (cents)	Assessment of linear data	Assessment of ZO data	
C–G	–3.2	–7.0	–3.8	A	A	shorter
G–D	–6.4	–3.9	2.5	A	A	
D–A	1.3	1.1	–0.2	A	A	
A–E	–21.3	4.9	26.2	C	D	
E–B	–12.4	3.0	15.4	B	A	
B–F#	–8.9	18.0	26.9	A	B	longer
F#–C#	–2.5	–2.0	0.5	A	A	
C#–G#	–3	–11.0	–8.0	A	B	little longer
G#–D#	–28.2	20.0	48.2	C	D	
D#–A#	–13.7	13.0	26.7	B	B	
A#–F	4	8.0	4.0	A	B	
F–C	–0.4	–2.0	–1.6	A	A	

fifth formed over A1, captured by the tuner, would be more affected by the change in the tension of the string, since it is located closest to the cursor at the fixation key.

Applying the deviation criteria with respect to the perfect fifth of the fifths calculated from the vibrating length of the string (A between –10 and 10; B < –10, and C < 20), we obtained the results shown in Table 10.

If we value the deviation (in cents) of the fifths obtained by measuring the frequencies with the Zen-On tuner with respect to the perfect fifth (Table 11), using the same criteria as those in the previous table, we obtain the results shown in Table 12.

The tuning system of Francisco Fernández's fortepiano

With the values that we obtained from the monochord, it can be deduced that the tuning system used in this instrument was a cyclical one based on unequal fifths. All fifths were practically shorter than the perfect fifth and the system was not a closed one, since the sum of the 12 fifths is not a multiple of the 1200 of an octave, but instead gives an interval of 8235.1 cents, with a deviation of –23.6 cents with respect to the multiple of the octave, practically equivalent to that of a comma (22 cents). This interval represents less than a quarter of a tone and was distributed among the 12 fifths, at –1.9 cents per fifth. This is a nearly negligible amount for musical purposes; in fact, if while tuning, we distributed this difference among all the fifths, we would have a closed system, in which the last note of the cycle is an exact enharmonic of the first. Among the tuning systems based on the irregular temperament of the fifths, those resembling ours most closely were the ones that derived from the mesotonics, in which some fifths were modified in order to close the circle.

From the measurements of the theoretical L2 of the monochord, we obtaind a tuning system based on eight fifths slightly different from perfect fifths, with three shorter ones and a longer one. But if we compare this system with the data from the tuner, we find six perfect fifths: four shorter and two considerably longer ones, as shown in Table 4.

All fifths that were formed with diatonic notes were similar in the two methods, except for A#–E#, which had already shown irregularities. The fifths formed with chromatic notes also followed a similar pattern, except for G#–D#, which differed by nearly a quarter of a tone, as in A–E. From these irregularities, two important questions arose: (a) Which of the two approaches is more reliable? (b) How reliable are the marks on the tuner, if the acoustic results differed from the theoretical ones, by up to 48.2 cents in some fifths, and could the differences be attributed to the inharmonicity of the strings? In any case, two important factors should be taken into account, as already stated: the inharmonicity of the string and the variation in tension at the left end of the monochord.

Finally, we decided to consider the measurements of the string as determined by the position of the cursor as the most precise, since we are studying the construction of the tuner. The tuning system that comes closest to the distribution of the theoretical fifths of the monochord is that of the 18th century French temperaments. These systems are derived from the mesotonic of one fourth of a comma, and in them some fifths are modified in order to close the circle. They are very similar to the tuning model suggested by Jean Philipe Rameau [10] in 1726, which gave way to that of Jean Jacques Rousseau [11] in 1767. In both, the first fifths, from Bb to B, are slightly shortened so that C–E and E–G# give a perfect major third.

Table 13. The monochord's deviations with respect to the perfect fifth

Fifths	Deviation (cents)
C–G	–10.0
G–D	–14.0
D–A	–6.9
A–E	–28.6
E–B	–20.4
B–F#	–17.4
F#–C#	–10.2
C#–G#	–11.2
G#–D#	–35.8
D#–A#	–21.8
A#–F	–4.5
F–C	–8.2

We applied a correction factor of +15 cents to the fifths of the monochord, deduced from the vibrating lengths, and compared the results to those of the French tuning of Rousseau. The intervals of the Francisco Fernández's monochord were notably closer to those of the method of Rousseau. Even so, two fifths, both of the wolf-tones type, more irregular than A–E and G#–D# were obtained (data not shown).

The results were so coherent with the tuning systems of the period when the fortepiano was made that they could be considered a true living model of the musical temperament of the Spanish keyboard instruments of the first half of the 19th century. However, it would be advisable to compare these systems with those of other organs and fretted string instruments that have been preserved, which would provide a wider and more diverse view of the harmonic and timbric sensitivity of musicians and music, one that is definitively immersed in the world of tonal chromatism and modulation. The general diapason of the instrument might have been deeper, circa 423 Hz, according to the unpublished studies by Òscar Laguna of the Avila organ tubes made by Garcimartín in 1828.

In conclusion, we suggest, for the keyboard instruments of the period that are still in use today, a tuning system based on the measurements and calculations obtained from the study of Francisco Fernández's monochord. If a procedure of successive fifths were followed, it would be necessary to start from the deviations with respect to the perfect fifths indicated in as follows (fifth/deviation/in cents): C–G/-10.0;G–D/-14.0;D–A/-6.9;A–E/-28.6;E–B/-20.4;etc.

From these values, proceeding with the necessary leaps of an octave for tuning, and adding the correction factor—approximately 5–10 cents per fifth (Table 3)—in accor-

Table 14. Monochord's deviations from the perfect fifth after applying the correction factors

Order	Fifths	Deviation from the perfect fifth (cents)	Deviation of the final application from the perfect fifth	Category	Practical deviation (cents)
1	A3–D3	–6.9	–1.9	A	0
2	D3–G2	–14.0	–4	A	0
3	G2–C2	–10.0	–10	B	–10
		Up 2 octaves			
4	C4–F3	–8.2	–3.2	A	0
5	F3–Bb2	–4.5	0	A	0
6	Bb2–Eb2	–21.8	–11.1	B	–10
		Up 2 octaves			
7	Eb4–Ab3	–35.8	–25.8	D	–20
8	G#3–C#3	–11.2	–10.2	B	–10
9	A3–E4	–28.6	–18.6	C	–15
10	E4–B4	–20.4	–10.4	B	–10
		Down 2 octaves			
11	B2–F#3	–17.4	–17.4	C	–15
12	F#3–C#3	–10.2	–10.2	B	–10

dance with the proximity of the measurement taken at the lower end of the monochord, the resulting values are those in Table 14.

Column 5 of the table is the result of a practical approximation to the deviations that must be measured with the tuning device, rounding them off with respect to the common values A, B, and C. In this proposal for a tuning system, two things are important: the order of operation in the tuning process from the keyboard, and the values of the deviations, in cents, to be applied to each fifth (column 5). Accordingly, a system is obtained in which four fifths can be considered perfect (A); five fifths will be tuned shorter, by some 10 cents (B), and two other fifths by some 15 cents (C); and one fifth will exceed this deviation, by 20 cents. This approach yields a tuning system based on the study of the monochord, valid for the aesthetics of the musical repertoires of the early 19th century, and especially for the pieces composed and interpreted on the Iberian Peninsula.

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